initial temperature of the test is not low and the volume coefficient of expansion is not small in comparison with the shock strength the first term on the right-hand side of (20) will dominate and the temperature rise will be approximately that given by (19).

To find a closer approximate solution to (20) we assume steady-state propagation of the wave as in (KG). Under this assumption, the momentum conservation equation reduces to the algebraic condition

$$\sigma = \sigma_h + \rho_0 V_0^2 (\rho_0/\rho - \rho_0/\rho_h). \tag{23}$$

Here, the value of σ_h will be determined by the specified material constitutive relation. When (10) and (23) are used to replace σ and p in (20), the post precursor temperature-rise is given by

$$
\begin{aligned}
\rho_0 \int_{T_h}^{T} C \, dT' = & -\tfrac{1}{2} K [\{\ln(\rho_0/\rho)\}^2 - \{\ln(\rho_0/\rho_h)\}^2] \\
& - K\beta T_0 [\ln(\rho_0/\rho) - \ln(\rho_0/\rho_h)] + \sigma_h [(\rho_0/\rho) - (\rho_0/\rho_h)] \\
& + \tfrac{1}{2} \rho_0 V_0^2 \{(\rho_0/\rho) - (\rho_0/\rho_h)\}^2 \\
& - (3/8G) \{(\rho_0/\rho)\sigma_h + \rho_0 V_0^2 (\rho_0/\rho)(\rho_0/\rho - \rho_0/\rho_h) + K\beta T_0 \theta - K \ln(\rho_0/\rho)\}^2 \\
& + (3G/2)\{\ln(\rho_0/\rho_h)\}^2.
\end{aligned}
\tag{24}
$$

To this we add the initial increase at the elastic wavefront to obtain the total temperature rise, viz. $\theta = (T - T_0)/T_0$, where

$$
\begin{aligned}
\rho_0 T_0 \int_0^{\theta} C \, d\theta' = & (K\beta T_0/\mu)[(\rho_h/\rho_0)^\mu - 1] \\
& - \tfrac{1}{2} K [\{\ln(\rho_0/\rho)\}^2 - \{\ln(\rho_0/\rho_h)\}^2] \\
& - K\beta T_0 [\ln(\rho_0/\rho_h)] + \sigma_h [(\rho_0/\rho) - (\rho_0/\rho_h)] \\
& + \tfrac{1}{2} \rho_0 V_0^2 \{(\rho_0/\rho) - (\rho_0/\rho_h)\}^2 \\
& - (3/8G) \{(\rho_0/\rho)\sigma_h + \rho_0 V_0^2 (\rho_0/\rho)(\rho_0/\rho - \rho_0/\rho_h) + K\beta T_0 \theta - K \ln(\rho_0/\rho)\}^2 \\
& + (3G/2)\{\ln(\rho_0/\rho_h)\}^2.
\end{aligned}
\tag{25}
$$

The important point about (25) is that it does not depend upon the constitutive relation which relates $\dot{\varepsilon}^p$ to the stress state, temperature, and other variables. Thus, the viscoplastic properties of the material have no influence on the temperature developed at any particular density level. The viscoplastic properties of the material of course play a dominant role in determining the spatial or temporal location of any particular pair of density and temperature values.

For constructing complete shock profiles by numerical integration (25) can be incorporated into the procedure with relative ease including, if known, the dependence of C on T. Here, since our purpose is to avoid such computations, the contribution of the term $K\beta T_0 \theta$ on the right-hand side of (25) is approximated using (19). Even for large values of β and large compressions this term is quite small compared with the additive term $K \ln(\rho/\rho_0)$ and, therefore, the approximation should be very accurate. For constant C, then we have the result that

$$
\begin{aligned}
\theta = & (\rho_h/\rho_0)^\mu - 1 \\
& + (\mu/\beta T_0)[\{(\sigma_h/K) + (\rho_0 V_0^2/2K)(\rho_0/\rho - \rho_0/\rho_h)\}(\rho_0/\rho - \rho_0/\rho_h) \\
& - \tfrac{1}{2}\{[\ln(\rho_0/\rho)]^2 - [\ln(\rho_0/\rho_h)]^2 - \beta T_0[\ln(\rho_0/\rho) - \ln(\rho_0/\rho) \\
& + (3K/8G)\{[(\rho_0\sigma_h/pK) + (\rho_0^2 V_0^2/pK)(\rho_0/\rho - \rho_0/\rho_h) - \ln(\rho_0/\rho) \\
& + \beta T_0(\rho/\rho_0)^\mu - \beta T_0]^2 - [(2G/K) \ln(\rho_0/\rho_h)]^2\}].
\end{aligned}
\tag{26}
$$

An alternative approach to estimating the temperature rise is to take (19) as describing the effect of adiabatic compression and then to linearize and approximate the plastic work terms. This leads to

$$\theta \approx [(\rho/\rho_0)^\mu - 1] + A\{\ln (\rho_0/\rho)\}^2, \tag{27}$$

where

$$A = (3K/8G)(\rho_0 V_0^2/K^* - 1)(1 + 4G/3K^* - \rho_0 V_0^2/K^*)(\mu + \mu/\beta T_0)$$

and

$$K^* = K(1 + \beta T_0 \mu).$$

The term in square brackets is due to adiabatic compression and the remaining term is heating due to plastic work. The limitations on the quantity $\rho_0 V_0^2$ were discussed in detail in (KG) and in the present context are $1 < \rho_0 V_0^2/\varkappa < 1 + 4G/3\varkappa$ where $\varkappa$ is the bulk compressibility.

In Figs. 1 and 2 we show examples of the differences among temperatures calculated on the bases of (19), (26) and (27) for numerical parameters set forth in Section 5. For a constant value of C, the simple power law underestimates the temperature in
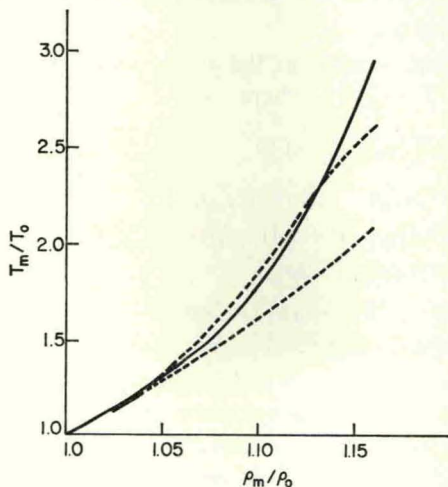

FIG. 1. Non-dimensionalized plot of maximum temperature versus maximum density for a family of shock waves. The solid curve is computed from (26) which is a nearly exact solution to the non-linear integral equation governing the temperature. The two dotted curves represent approximate solutions given by (19) and (27). Material characteristics used in the computations are $G = 3K/8$, $\mu = 5$, $\beta T_0 = 10^{-2}$ and $\rho_h = \rho_0$.

comparison to the more exact equation (26). For relatively weak shocks, addition of the plastic work term that occurs in (27) improves the agreement at small strains with some overcompensation at larger strains. For stronger shocks, (27) underestimates the temperature in comparison with (26) at all strain values. On the other hand, if any variation of C is to be taken into account (25) must be used to determine the temperature rise.

The dependence of pressure on density and temperature, (10), can be approximated at small strains using (19) which gives

$$(\rho_0/\rho)(p/K) = \ln (\rho/\rho_0) + \beta T_0[(\rho/\rho_0)^\mu - 1]. \tag{28}$$